1. Problem sheet for Set Theory, Winter 2012

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Let x, y, z be variables and A a class term.

Problem 1. Prove

a) $\forall x \forall y \exists z \; z = (x, y)$.

b) If $x, y \in A$, then $\{x\}, \{x, y\} \in \mathcal{P}(A)$ and $(x, y) \in \mathcal{P}(\mathcal{P}(A))$.

c) If $(x, y) \in A$, then $x, y \in \bigcup \bigcup A$.

Problem 2. Let $\neg\{x \mid \varphi(x)\} := \{x \mid \neg\varphi(x)\}$ for $\varphi \in L^{\in}$, $0_V := \{x \mid x \neq x\}$, and $1_V := \{x \mid x = x\}$. Suppose u, v, w are class terms of the form $\{x \mid \varphi(x)\}$, where $\varphi \in L^{\in}$ contains no free variables except for x. Show that

a) $u \cup v = v \cup u$.

b) $u \cup (v \cup w) = (u \cup v) \cup w$.

c) $u \cap (v \cup w) = (u \cap v) \cup (u \cap w)$.

d) $u \cap (u \cup v) = u$.

e) $u \cup (\neg u) = 1_V$.

f) $u \cap (\neg u) = 0_V$.

Since the equations a) - d) with $\cup$, $\cap$ exchanged can be checked analogously, these terms for a Boolean algebra.

There are 6 points for each problem. Please hand in your solutions on Monday, October 15 before the lecture.